

Dayalan P. Kasilingam and David B. Rutledge  
 Department of Electrical Engineering  
 California Institute of Technology  
 Pasadena, California 91125

**Abstract.** Coplanar transmission lines lose energy to surface waves when the propagation constant of the surface-wave mode exceeds that of the transmission line. This happens when the substrate thickness is an appreciable fraction of a wavelength. The losses should become important in integrated circuits at near-millimeter wavelengths because it is hard to make the substrate thickness small compared to a wavelength. In this paper we have developed a theory based on reciprocity for predicting these losses. We also utilized the quasi-static approximation method to derive expressions for propagation constants and line impedances. Experimental measurements were made for the surface-wave losses in the two strip line, the two slot line and the three wire line, and the results obtained were consistent with the theory.

Integrated circuits are now being fabricated at near-millimeter wavelengths.<sup>1-4</sup> Coplanar transmission lines (Fig. 1) are often the most convenient guides at these frequencies, but it can be hard to make the substrate thickness small compared to a wavelength. This means that the lines will radiate into substrate modes, in contrast to the way these lines behave at microwave frequencies. Much information is available on other engineering characteristics of these lines,<sup>5,6</sup> but not on surface-wave losses. Understanding these losses is important in designing integrated-circuits at these wavelengths, not only to minimize losses, but also because this radiation could be useful. In this paper we study the losses of various transmission lines on different substrates.

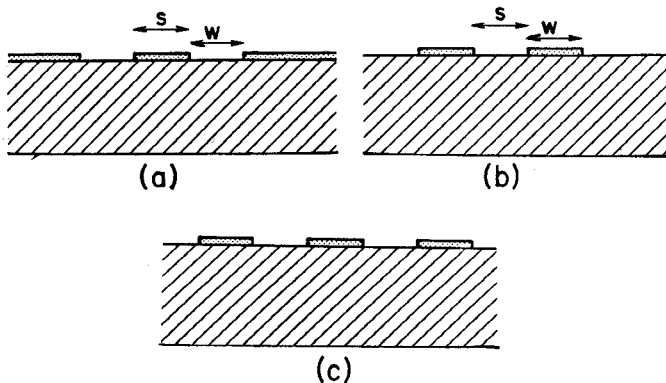


Fig. 1 Coplanar Transmission Lines: (a) two-slot line (b) two-strip line (c) three wire line.

It is easy to understand how this radiation occurs. At an interface, waves propagate along metals at a velocity that is intermediate between the velocity of light in air and the velocity of light in the dielectric. As far as the dielectric is concerned,

the wave is fast, and radiation will occur in the dielectric in the manner of a leaky wave antenna. The radiation peak is at an angle  $\psi$ , given by

$$\cos\psi = k_z/\beta \quad (1)$$

where  $k_z$  is the guide propagation constant along the line and  $\beta$  is the propagation constant of the substrate mode. This is illustrated in Fig. 2a. In a thick substrate radiation is in a semi-cone of angle  $\psi$  (Fig. 2b). This formula indicates that the

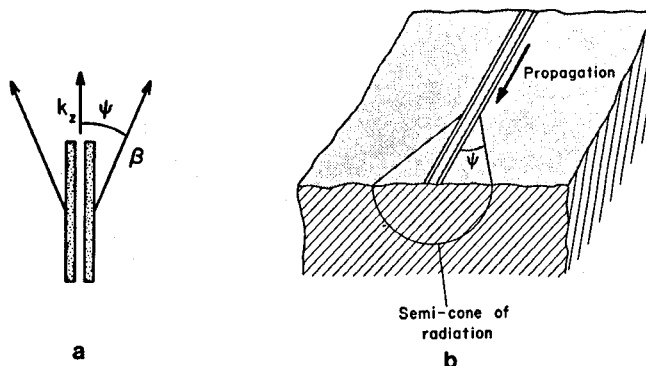


Fig. 2 Radiation by coplanar transmission lines. (a) into surface-wave modes (b) into a thick substrate in a semi-cone of angle  $\psi$ .

line will begin to suffer losses to a particular mode when the mode propagation constant  $\beta$  exceeds the guide propagation constant  $k_z$ . Each mode has a frequency at which  $\beta$  first exceeds  $k_z$  and this turn-on frequency is somewhat larger than the usual cut-off frequency. This is shown for a two strip line in Fig. 3. The  $TE_1$  and  $TM_1$  modes' cut-off thickness is indicated by the broken line. Also shown on the figure are the experimentally measured values of  $k_z$ . The radiation patterns of these surface waves can be observed by putting the transmission line on a quadrant of a circular disk (Fig. 4). The edge is tapered to prevent reflections. A vertically polarized receiving horn picks up the TM waves and a horizontally polarized horn picks up the TE waves. Typical surface-wave radiation patterns for a coplanar two strips transmission line and a coplanar slot line are shown in Fig. (5). Surface wave radiation patterns for elementary dipoles and slots<sup>7</sup> have also been derived from reciprocity.

We have recently developed a theory, based on reciprocity and the quasi-static approximation method which gives the surface-wave attenuation constant of coplanar transmission lines. In all cases we find simple formulas with losses proportional to the inverse of the effective guide thickness  $h_e$ . The effective guide thickness is a term from the ray picture of waveguide propagation in integrated optics,

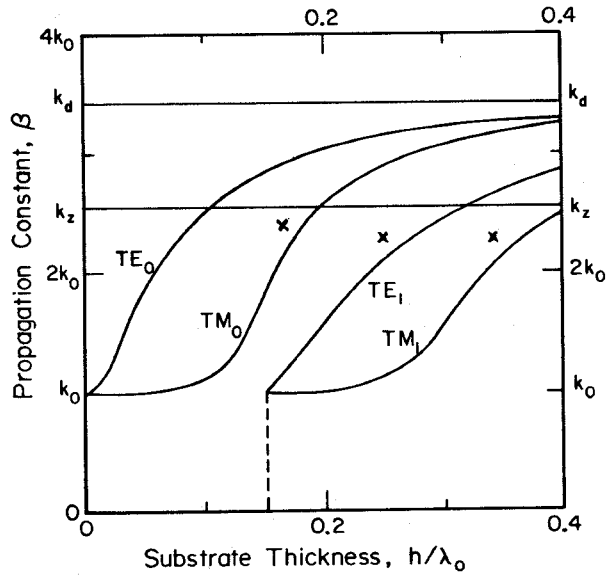


Fig. 3 Dispersion curves for surface-waves as a function of thickness with  $\epsilon_r = 12$ .  $k_d$  and  $k_z$  are the propagation constants in the dielectric and the line respectively.

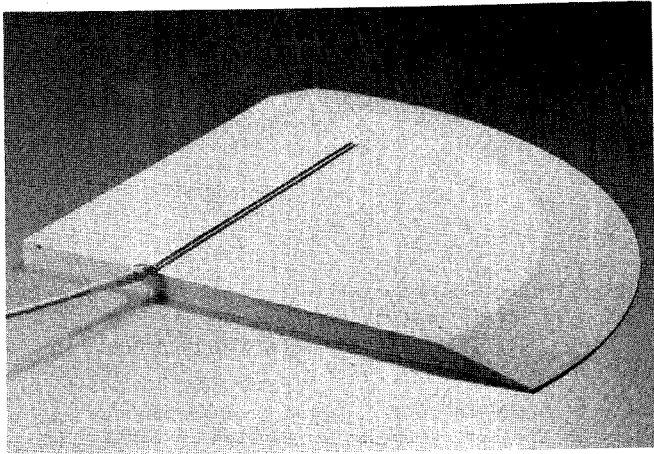


Fig. 4 Disk for measuring surface-wave losses for a two strip line.

and is given by the actual thickness plus the apparent penetration depth of the ray (Fig. 6). At high frequencies the effective thickness tends towards the physical thickness of the substrate. The quasi-static approximation<sup>8</sup> enables us to derive analytic expressions for the characteristic impedances of the coplanar transmission lines. It also gives estimates for the propagation constants. The attenuation constant due to TM and TE modes in a two slot line are as following

$$\alpha_{TM} = \frac{149.6}{\sqrt{1 + 1/\epsilon_r}} \frac{\sin^3 \psi \sin \theta_d}{h_e K K'} \left(\frac{W}{\lambda_d}\right)^2 \text{ dB/m} \quad (2)$$

$$\alpha_{TE} = \frac{149.6}{\sqrt{1 + 1/\epsilon_r}} \frac{\sin 4 \cos^2 \psi \sin \theta_d \cos^2 \theta_d}{h_e K K'} \left(\frac{W}{\lambda_d}\right)^2 \text{ dB/m} \quad (3)$$

where  $\psi$  and  $\theta_d$  are those shown in Fig. 2a and Fig. 6.  $W$  is the width of the slot and  $K, K'$  are elliptic integrals defined by Wen.<sup>9</sup> Fig. 7 show the losses as

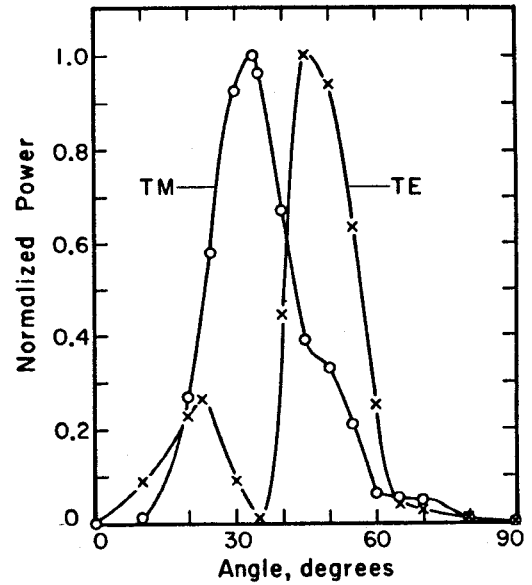


Fig. 5 Measured radiation pattern for a two strip line (strip width 2mm, strip spacing 1 mm, 5 GHz, actual thickness is 12.5 mm,  $\epsilon_r = 12$ ).

a function of width for both two slot line and two strip line on different dielectrics. Fig. 8a and Fig. 8b show the frequency dependences of the losses for a two slot line and a two strip line, respectively.

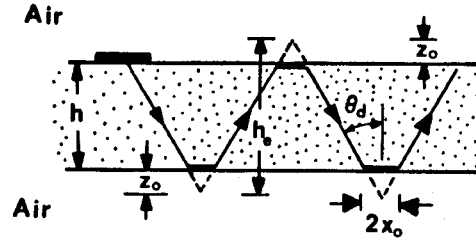


Fig. 6 Effective thickness: ray picture,  $h_e = h + 2z_0$ .

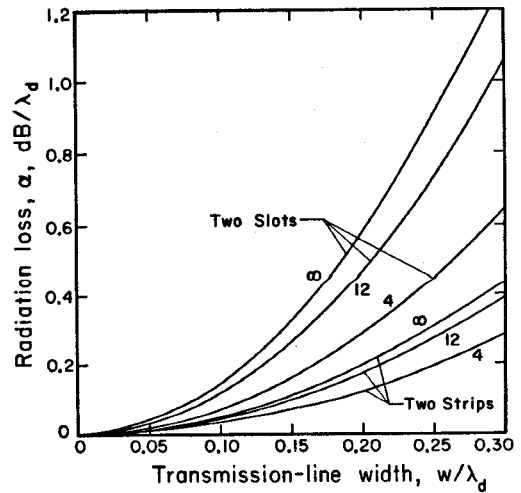


Fig. 7 Theoretical losses for lines with a strip or slot spacing equal to 1/3 the total width. The dielectric constants are indicated. The substrate thickness is assumed to be much greater than  $\lambda_d$ .

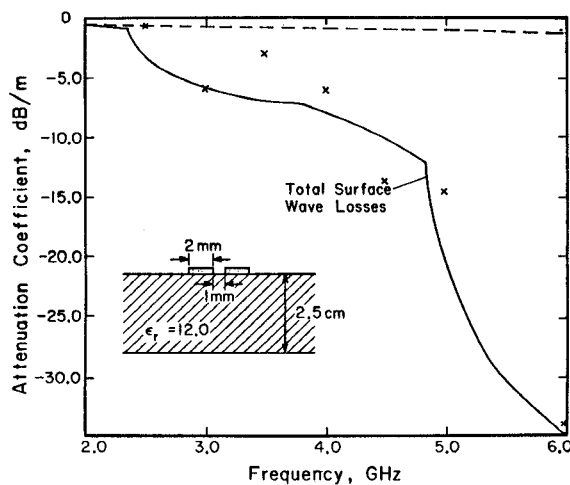
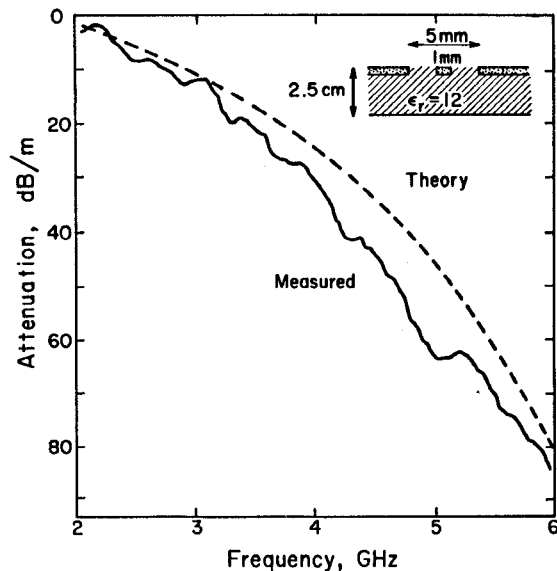


Fig. 8 (a) Measured losses, compared to theory for a two slot line.  
(b) Measured losses (x) and theoretical losses for two strip line. Broken line indicates conduction losses.

In all the cases the attenuation constant  $\alpha$  was calculated utilizing the equation

$$\alpha = \frac{\text{Power loss/unit length}}{2(\text{Power transmitted})} \quad (4)$$

The power loss per unit length was calculated using reciprocity.<sup>10</sup> In effect we relate the surface wave power per unit length to the line voltage (or current). The transmitted power may also be represented in terms of voltage (or current) and the characteristic impedance. The characteristic impedances is found by the quasistatic approximation method, and are given by

$$Z_{\text{two strip}} = \frac{K(k)}{K'(k)} \frac{2}{\epsilon_r + 1} \eta_0$$

$$Z_{\text{two slot}} = \frac{K'(k)}{K(k)} \frac{2}{\epsilon_r + 1} \eta_0$$

where  $\eta_0$  is the free space impedance,  $\epsilon_r$  the dielectric constant, and from Fig. 1.

$$k = \frac{S}{S+2W}$$

As shown in Fig. 3, the measured values of the propagation constant were found to be smaller than their quasi-static approximation, by 5-10 percent. This means the wave in the transmission line propagates at a velocity faster than the phase velocity predicted by the quasi-static approximation. A plausible explanation for this discrepancy is yet to be found. Davies<sup>11</sup> et. al. show that the propagation constant and impedance of the coplanar transmission line are not equal to their quasi-static values when the substrate is of finite thickness, but the values we measured still do not agree with the modified values they obtained. Kitazawa<sup>12</sup> et al have predicted that the propagation constant decreases with increases in metal thickness. Even this premise does not explain the magnitude of our discrepancy.

The conduction losses for the two strip line are also shown in Fig. 8b. These losses are comparatively small in the S band. They remain small even at near-millimeter frequencies although the surface resistance does increase with frequency. At 90 GHz, the conduction loss attenuation constant,  $\alpha_c$  is approximately 45 dB/m for the line we experimented with.

We have in this paper tried to quantify the effects of surface waves in coplanar transmission lines. The most significant conclusion is that at frequencies above the "turn on" frequencies the surface wave losses dominate all other loss mechanisms, up to submillimeter frequencies. The theory derived for surface wave losses also apply to lines at near-millimeter wavelengths and the losses are therefore indicative of the losses one would experience at the higher frequencies. We have also critically examined the quasi-static approximation method used frequently in micro strip line and coplanar transmission line analysis.

#### Acknowledgements

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